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Assessment and Instruction for Mathematics Proficiency: Implications for State Systems

SECC English as a Second Language Institute—Jazzin' It Up: A Medley of Notes for Creating a Culture That Supports English Language Learners

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Session Questions

- What is mathematics proficiency?
- Do our assessments measure mathematics proficiency?
- How do we assure that we measure for mathematics proficiency?
- What does the research say about measuring English language learners' (ELLs') mathematics proficiency?
- What are the implications for state systems and district/schools in designing assessments and instruction for ELLs' mathematics proficiency?

What is Mathematics Proficiency?

- Conceptual understanding
- Procedural fluency
- Strategic competency
- Adaptive reasoning
- Productive disposition
  (Effort versus Ability, Math Panel Report)

Source: National Research Council (2001)

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How do we assess Mathematics Proficiency?

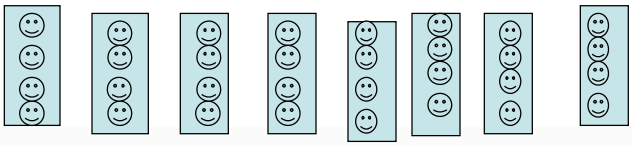
Let's consider the Smiley Face Cookie Task

A second grade teacher presented this scenario:

- A bakery makes Smiley Face cookies.
- The cookies are sold 4 to a package.
- How many Smiley Face cookies are there in 8 packages?

A Student's Answer

A second-grade student turned in this written assignment for which she drew 8 packages with 4 Smiley Face cookies in each, along with the handwritten numbers below each package as shown. At the end she wrote her final answer.



4 8 12 16 20 24 28 32

Answer: *There are 32 Smiley Face cookies.*

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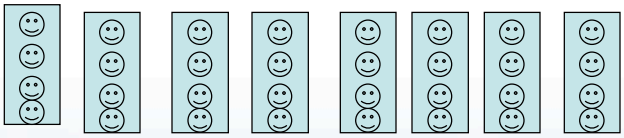
Discussion

- Talk to someone about the following: Given this information, what would you say about this second grader's mathematics proficiency?
- Be ready to share salient points about your discussion in 3 minutes.

The Real Story

When asked by the teacher to describe her strategy, she indicated that after she had drawn her 8 packages that she had counted the cookies in the first package by ones and then wrote her answer 4. She started the next package by beginning her count with 5, etc.



4 8 12 16 20 24 28 32

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What are these items assessing?

Review the fourth-grade assessments in the next four slides.

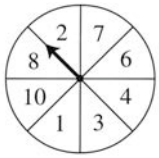
Discuss your ideas with two other participants.

Rank them first by how well they assess mathematics proficiency. Then rank them by the effect these items might have on ELLs taking these assessments.

Write your ideas on chart paper, and be ready to share in 15 minutes.

Word Problem Example 1

To win a game, Tamika must spin an even number on a spinner identical to the one shown below.



Are Tamika's chances of spinning an even number certain, likely, unlikely, or impossible?

A. certain
B. likely *
C. unlikely
D. impossible

Source: Massachusetts Department of Education (2003)

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Word Problem Example 2

Which of the following is read as "five-tenths"?

A. 0.05
B. 0.5*
C. 5.10
D. 5.0

Source: Massachusetts Department of Education (2003)

Word Problem Example 3

Mr. Garcia gave each of his students a notepad, a pencil, and a ruler on the first day of school. The chart below shows the different colors of the notepads, pencils, and rulers.

Notepads	Pencils	Rulers
Yellow White	Red Blue Orange	Green Purple

How many different combinations of 1 notepad, 1 pencil, and 1 ruler can Mr. Garcia make? (Answer: 12)

Source: Massachusetts Department of Education (2003)

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Word Problem Example 4

Every Saturday in the fall, Martin has to do 1 inside chore and 1 outside chore. The chores are listed below. How many different combinations of 1 inside chore and 1 outside chore can Martin make?

Inside Chores	Outside Chores
vacuum wash dishes dust	rake weed

A. 3
B. 5
C. 6 *
D. 9

Source: Massachusetts Department of Education (2003)

Maria Martiniello's Study of the Linguistic Complexity of Mathematics Word Problems

- The study included ELLs and non-ELLs with equivalent mathematics proficiency taking a statewide fourth-grade mathematics test.
- The study builds on prior research showing that the greater linguistic complexity increases the difficulty of English mathematics items for ELLs compared to non-ELLs of equivalent mathematics proficiency.
- According to the researcher, the relationship between some of the linguistic features and the differential difficulty of of math word problems for ELLs and non-ELLs has been investigated in elementary, middle, and high school students.
- Language proficiency status was highly associated with socio-economic status in this study.

Source: Martiniello, M. (2008)

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Differential Item Functioning (DIF)

Definition of DIF: "discrepancies in difficulty an item presents for members of two groups who have equivalent levels of proficiency on the construct the test is intended to measure."

Source: Martiniello, M. (2008)

Methods of the Study

- Analyzed the linguistic complexity of six mathematics test items and used think-aloud protocols to gather evidence of comprehension difficulty for Spanish-speaking ELLs.
- Used two DIF methods: evidence disfavoring ELLs and the degree of DIF.
- Also identified the learning strand, the linguistic complexity, and children's responses to the mathematics test items in the think-aloud interviews.

Source: Martiniello, M. (2008)

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Measures

- Used a detailed microanalysis of the text's syntactic complexity for each item.
- Used expert ratings of the items' overall syntactic and lexical complexity.
- Cross-checked items' vocabulary with *A List of 3000 Words Known by Students in Grade 4* and *Living Word Vocabulary.*
- Conducted think-aloud interviews individually, noting decoding errors, recording the types of mistakes children made.

Source: Martiniello, M. (2008)

Linguistic Features of DIF Disfavoring ELLs

- Syntax:
  - multiple clauses
  - long noun phrases
  - limited syntactic transparency
- Vocabulary:
  - unfamiliar to grade level
  - context
  - polysemous words
  - less familiar cognates
  - English words and expression that signify particular referents of mainstream America

Source: Martiniello, M. (2008)

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Implications for State Accountability and Assessment Systems

- Discuss the following in groups of three:
  What are the implications of the study (and other similar studies) for the following:
  - Item writers
  - The use of DIF
  - The use of think-alouds
  - The use of test scores to make inferences about ELLs' mathematics proficiency
  - Other
- Be ready to share salient points of your discussion in 10 minutes.

Recommendations from the Center on Instruction

- ELLs need early explicit and intensive instruction and intervention in basic mathematical concepts and skills.
- Academic language is as central to mathematics as it is to other academic areas. It is a significant source of difficulty for many ELLs who struggle with mathematics.
- ELLs need academic language support to understand and solve the word problems that are often used for mathematics assessment and instruction.

Source: Francis, D. F., Lesauz, N., & Rivers, H. (2006)

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Language and Mathematics:
Implications for the Mathematics Instruction of English Language Learners

Recommendation from the Center on Instruction

Academic language is as central to mathematics as it is to other academic areas. It is a significant source of difficulty for many ELLs who struggle with mathematics.

Source: Francis et al. (2006)

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Examining the Convergence of Mathematics and Language

- Imagine you are junior high school student taking the state exit exam and you come upon problem number 14. Solve it and discuss with others at your table.

- 14) Find the *ugloft* of a *bipkad* if the *rexnuza* is 20.

Examining the Convergence of Mathematics and Language (cont.)

Find the *ugloft* of a *bipkad* if the *rexnuza* is 20.

You look on the information sheet of the TAKS test and see the following:

ugloft = area
bipkad = circle
rexnuza = diameter

Now solve the problem. What made the difference?

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Language and Mathematics

- Mastery of academic language is arguably the single most important determinant of academic success for individual students. . .Proficient use of—and control over—academic language is the key to content-area learning.

Source: Francis, et al., p. 7

Connecting Mathematics and Language

Academic language in mathematics extends far beyond numbers. Academic language in mathematics includes not only its own vocabulary, but is also further complicated by the extensive use of symbolic and graphical representation.

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CONNECTING MATHEMATICS AND LANGUAGE

- **The mastery of math concepts presupposes facility with the academic language used to characterize, express, and apply concepts, yet in math classrooms and curricula across the U.S., ELLs don't understand much of the language that is used, and most learners are not explicitly taught to read, write, or speak mathematically.**
- **Source: Lager, C. A. (2006)**

Connecting Mathematics and Language (cont.)

In math classrooms and curricula, the language demands are likely to go unnoticed and unattended to. This lack of attention occurs despite the permeation of academic language through much of the curriculum as well as standards-based tests.

Source: Francis, et al., p. 37

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Connecting Mathematics and Language (cont.)

Every teacher must incorporate into his or her curriculum instructional support for oral and written language as it relates to the mathematics standards and content. It is not possible to separate the content of mathematics from the language in which it is discussed and taught.

Source: Francis, et al., p. 38

Connecting Mathematics and Language (cont.)

The convergence of language and mathematics is not an ELL problem (ELL students).

It is an ALL problem (ALL students)!

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Reality: Mathematics and Language

Research on teachers' perceptions has found some contradictions in the way teachers conceive of mathematics instruction (as free from language) and the kind of math assessments they use in their classrooms (with great language demands).

Source: Bunch, G., Aguirre, J., Tellez, K., Gutierrez, R., & Wilson, J. (2007)

Reality: Mathematics and Language (cont.)

The inattention to language in mathematics education applies to both assessment and instruction. In addition, problems in mathematical instruction due to language are further exacerbated by issues around not only *academic* language but also the *instructional* language used by teachers.

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Academic Language Issues

A tradition that has evolved in our education system is that mathematics class is for mathematics and English class is for English. This results in these two academic areas as mutually exclusive with ELLs in particular being shortchanged.

Academic Language Issues (cont.)

- When is the last time that you had hall duty and overheard two students walking by discussing integers, or inverse functions, or equivalent fractions?
- The fact that mathematical terms are rarely used outside of the mathematics classroom obviously restricts the acquisition of a deep and meaningful mathematics vocabulary.

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Academic Language Issues (cont.)

High-frequency words are more likely to be recognized and decoded fluently, while low-frequency words are less likely to be recognized, thus slowing down the reading process, increasing the memory load, and interfering with text comprehension.

Source: Martiniello, M. (2008)

Academic Language Issues (cont.)

The English language is itself a complex language with exceptions to most rules coupled with terms that are abstract, ambiguous, and polysemous (words with multiple meanings).

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Difficulty of the English Language

Whose idea was this?

Difficulty of the English Language (cont.)

Reduce:

$$\frac{6}{8}$$

Answer: $\frac{6}{8}$

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Difficulty of the English Language (cont.)

Which is larger?

- **37**

5

Difficulty of the English Language (cont.)

- Part of the difficulties occur because of polysemous words whose multiple meanings can cause confusion.
- Polysemous words can have multiple meanings within mathematics or have one meaning in mathematics and another in standard English.

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Difficulty of the English Language (cont.)

Examples of polysemous words within mathematics:

- Ounce—Weight or volume?
- A “number” can still be about mathematics but not refer to a quantity (i.e., number as a name, code, or location).
  - 512-476-6861
  - 30218 Main Street

Difficulty of the English Language (cont.)

- Mathematical terms such as number can have multiple meanings *outside* of mathematics.
- The opening number was the highlight of the show.
- His last girlfriend really did a number on him.
- My finger is number than it was 5 minutes ago.

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Difficulty of the English Language (cont.)

Other issues can further cloud the academic language landscape. For example, a definition for a mathematical term such as an exponent may suffice at the elementary or middle school level, but the definition must be changed or refined in higher level mathematics.

Example: ($7^{1/2}$)

Traditional Language and Instruction

Traditional instruction:

- Does not sufficiently address mathematical language
- Often is not cognizant of the critical role of symbols and representation

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Traditional Language and Instruction (cont.)

- Traditional instruction pays little attention to mathematics language and symbolism and their impact on learning. In many ways, traditional instruction adds to the language problem.
- Consequently, we are perpetuating some of the language-based problems in mathematics instruction.

Traditional Language and Instruction (cont.)

- Research indicates that many educators experienced “traditional instruction” and tend to teach in the same way as they were taught.
- This tendency to teach as we were taught includes the associated language and symbols.

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The Instructional Language of Mathematics

The language that was used to teach today's teachers in their K–12 experience is like a baton that gets passed along. Unless changed by college pre-service training or subsequent professional development, that mathematics register becomes the instructional language of the next generation of teachers.

The Instructional Language of Mathematics (cont.)

Teachers' tendency to use the same terms that were used when they learned mathematics includes "careless" vocabulary using terms that have other meanings in standard English.

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The Instructional Language of Mathematics (cont.)

Discuss with others at your table what is problematic with the following (for both ELL and ALL students)!

a) Give your little brother the **"*bigger half*"** of the cookie

b) **"*Carry*"** the one

c) ***"Borrow"*** a ten

The Instructional Language of Mathematics (cont.)

Discuss with others at your table what is problematic with the following:

d) "*Cancel*" when simplifying a fraction or rational expression

e) "*Reduce*" a fraction

f) Two **"*goes into*"** eight

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The Instructional Language of Mathematics (cont.)

- The careless use of words is compounded further by misguided pronunciation:
- a) What are some ways to state 2632?
  - two thousand six hundred and thirty two
  - Twenty six hundred thirty two
  - Twenty six thirty two
- How can this be problematic?

The Instructional Language of Mathematics (cont.)

b) 2560 and $25.60 – How are these often pronounced, and what is problematic about it?

c) 25.6 – How is this often pronounced, and what is problematic about it?

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The Instructional Language of Mathematics (cont.)

Mathematics language is sometimes made even more difficult because of "fluid" mathematical terms. . .someone somehow invents new names or changes meanings in mathematics terminology.

Examples:
Definition of trapezoid
-(-x) = x is the Op-Op property
Commutative property = "Order property"
"FOIL" Method
New definition of "average"

The Instructional Language of Mathematics (cont.)

- Consider this problem:

$$\frac{10}{1/2} =$$

- How much sense does the question *"How many times does 1/2 go into 10?"* make?

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Addressing Language in Mathematics Instruction

- What can be done to improve instruction in mathematics so that language is addressed?
- A "fruity" analogy

Addressing Language in Mathematics Instruction (cont.)

Although not apparent on the surface, teachers can assist ELLs by teaching for deep conceptual understanding. Conceptual understanding enables students to accomplish the following:

1) View an idea/concept from multiple perspectives

2) Make critical connections to other fundamental concepts/ideas

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Addressing Language in Mathematics Instruction (cont.)

3) Recognize and understand subtleties in language, symbolism, and representation.

The above ability is key for ELLs. Conceptual understanding enables students not only to decipher the meanings of words in mathematics from the meaning in standard English, but also to decipher the subtle meanings of words in different mathematical contexts.

Addressing Language in Mathematics Instruction (cont.)

Teachers should use the ambiguity and multiple meanings of polysemous words as an asset in instruction rather than allow it to be a liability.

Example 1: Are a square and a rectangle similar?

Example 2: One pig grew from 5 pounds to 10 pounds. Another grew from 100 to 108 pounds. Which pig grew more?

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Addressing Language in Mathematics Instruction (cont.)

Another key component is to insure that instruction include students' interpretation of mathematical symbolism.

Example: Write the following problem out in words.

$3 < x < 8$ _____

Addressing Language in Mathematics Instruction (cont.)

- Student: "Miss, do you know what time it is?
- Teacher: Glances at watch and responds, "Yes, I do!"

- Lesson: Instruction in mathematics requires clarity and precision with the words that are used.

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Addressing Language in Mathematics Instruction (cont.)

- Unfortunately, mathematical language is usually slighted in state standards.

- Teachers must compensate and make language an integral focus and part of mathematics instruction and assessment.

Addressing Language in Mathematics Instruction (cont.)

Mathematics is not just numbers and computation. Teachers must keep in mind that, more often than not, the trouble with math is English!

QUESTIONS?

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