

## AFTERSCHOOL TRAINING TOOLKIT

### Math Tools

#### Cell Phone Plans (Teacher’s Guide)

The Bell sisters, Ding and Ling, are comparing cell phone plans. They want to share a plan and need to decide which option is the best for them. Here are two monthly plans that are available in their area:

- **Plan A:** \$30 flat fee per month and \$0.10/minute for each call (a “pay as you go” plan);
- **Plan B:** \$60 flat fee per month (for up to 900 minutes);

Help the Bell sisters decide which plan is best for them.

Begin by completing the table:

	Cost	
Minutes	Plan A	Plan B
0	30	60
100	40	60
200	50	60
300	60	60
400	70	60
500	80	60
600	90	60
700	100	60
800	110	60
900	120	60
Any number of minutes, $N$	$30+0.10N$	60

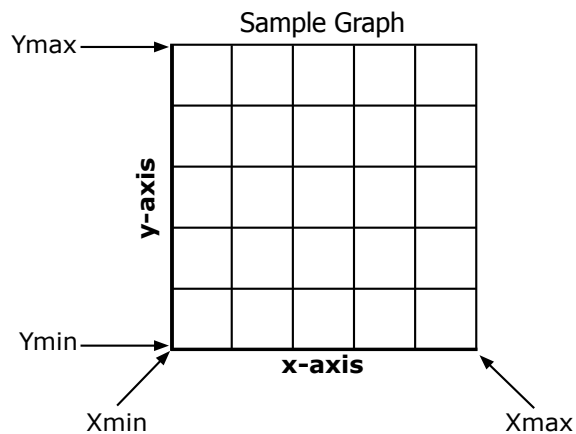
Some students may have difficulty filling in the final row of the table. You can help them by asking how they might find various numbers not listed on the table. For instance, how would they find the cost for 750 minutes? Try to get students to articulate that they would multiply the number of minutes by \$0.10, and then add \$30. This will help students translate to the more abstract expression using a variable. They might be confused because there is not a variable in the expression for Plan B. Help students see that the cost is always \$60. The cost does not change, and thus there is not a variable in the expression.

1. Using graph paper, plot the points for each plan on the same set of axes. Use a different symbol to mark each plan, and make a key to show which symbol goes with which plan.

Provide graph paper for students. Students should label the axes and create an appropriate scale for each axis. The x-axis should be labeled as minutes, and could be scaled from 0 to 1000, in 100-minute increments (or something similar). The y-axis should be labeled dollars (or cost), and could be scaled from 0 to 150, in \$10 increments (or something similar). Students refer back to this hand-drawn graph when they work on the upcoming graphing calculator questions. It is important for students to make as many connections as possible between the hand-drawn graph and what they see and enter on the calculator screen.

This is also a good opportunity to talk to students about scale, and independent and dependent variables. In this case, the independent variable is minutes (along the x-axis) and the dependent variable is cost (along the y-axis). The cost depends on the number of minutes one uses.

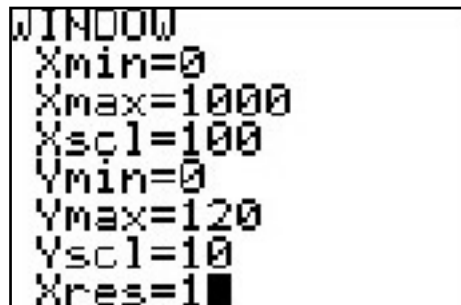
2. Enter the equations you found for each plan into the graphing calculator by pressing the  $\boxed{Y=}$  button. Then graph the equations by pressing  $\boxed{\text{GRAPH}}$ .
  - a. Hint #1: Substitute X for N from your expression in the last row of the table. Press  $\boxed{X,T,n}$  to display the X in your equation.
  - b. Hint #2: Make sure to set an appropriate window for your graph. Press  $\boxed{\text{WINDOW}}$  to set the parameters. Use the same scale from the graph you made in Question 1 for the Xmin, Xmax, Ymin, and Ymax (set Xscl=100, Yscl=10, and Xres=1).



Here is a screen shot of the equations entered after pressing **Y=** :



Be sure to set an appropriate window. Press **WINDOW** and set the following parameters. We want to see the graph from  $x = 0$  to 1000 minutes, and  $y = 0$  to 120 dollars:



- There are two  $y$ -intercepts on the graph. Name the values of the two  $y$ -intercepts. What do the  $y$ -intercepts represent in terms of the problem?

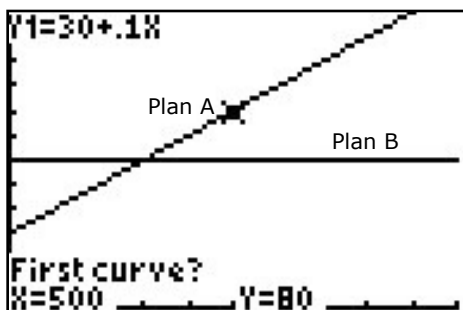
**Plan A:**  $y$ -intercept is 30. This represents the base rate of \$30 required every month, no matter how many minutes are used during the month.

**Plan B:**  $y$ -intercept is 60. This represents the base rate of \$60 required every month, no matter how many minutes are used during the month.

- The two graphs come together at a point of intersection. What is the point of intersection? You can use the **CALC** button on the graphing calculator to find the exact values. Choose 5: intersect.

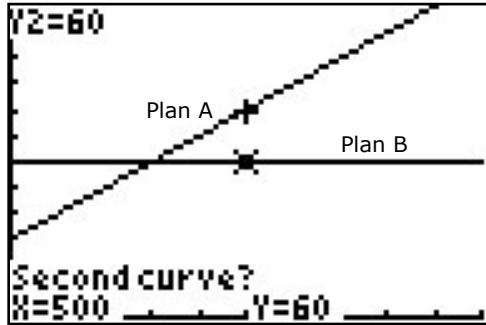
### Directions and screen shots from the TI-83 Plus Silver Edition

- After entering the equations in the calculator and setting an appropriate window, press **2<sup>nd</sup> TRACE**, and choose 5: *intersect*;
- The current graph is displayed with *First Curve?* in the bottom left-hand corner (Note: *Plan A* and *Plan B* added for clarification. This will NOT show up on the calculator. Also note the equation in the top left-hand corner tells you what graph the cursor is on):



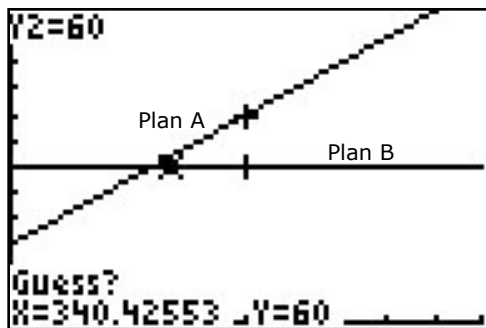
Press the up or down arrow, if necessary, to move the cursor to the first function and press **ENTER**;

3. The current graph is displayed with *Second Curve?* in the bottom left-hand corner:

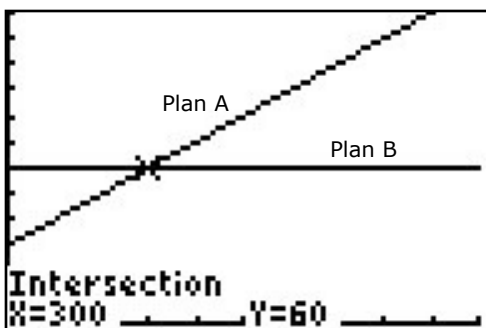


4. Press the up or down arrow, if necessary, to move the cursor to the third function (because we want the intersection between Y1 and Y3) and press **ENTER**;

5. Press the right or left arrow key, if necessary, to position the cursor close to the point of intersection, and press **ENTER**:



6. The calculator places the cursor on the point of intersection, and the x- and y-values of the intersection are displayed along the bottom of the screen:



5. Describe what the point of intersection represents in terms of the problem.  
This point of intersection (above) tells us that when  $x = 300$  minutes, the costs of Plans A and B are equal, at \$60/month.

6. Is there a way to use the table to determine the intersection point on the graph? If so, how? You can use the **TABLE** button on the graphing calculator to view the table. You can adjust the table by pressing **TBLSET**, above the **WINDOW** button. Yes, the table can be very helpful in locating points of intersection. Look for a row with the same value in each of the Cost columns. For example, at 300 minutes, both Plan A and Plan B cost \$60.

The graphing calculator has a table function that allows the user to see values for each function. Press **2<sup>nd</sup>GRAPH** to see the table. To set the table starting point and intervals, press **2<sup>nd</sup>WINDOW**. You will see the screen shot below (left). Enter the starting point in TblStart and the interval in Tbl. Set Indpnt and Depend to Auto. After you set the starting values, press **2<sup>nd</sup>GRAPH** again to see the table.

Scroll through the table screenshot (right), looking for a row where Y1 and Y2 have the same values. Notice that when  $X = 300$ , both Y1 (Plan A) and Y2 (Plan B) equal \$60. This is the point of intersection between the two functions.

TABLE SETUP		
TblStart=	0	
ΔTbl=	100	
Indpnt:	Auto	Ask
Depend:	Auto	Ask

X	Y1	Y2
0	60	30
100	60	40
200	60	50
300	60	60
400	60	70
500	60	80
600	60	90

X=0

7. Help the Bell sisters by writing a recommendation for choosing a phone plan. Use data from your graph and table to support your recommendation. Look for students to explain when and/or why Plan A would be advisable (e.g., when the number of minutes per month is  $< 300$ ) and when and/or why Plan B would be advisable (e.g., when the number of minutes per month is  $> 300$ , because it is worth it to pay a little more initially in case you go over 300 minutes per month). Ask questions to help students analyze this scenario, like, "What do you see in the graph/table that leads you to this conclusion?"

### Extension Activities

The Bell sisters find another plan they wish to include in their analysis. They call it **Plan C**. Here are the three plans they are interested in (**Plan A** and **Plan B** haven't changed):

- **Plan A:** \$30 flat fee per month and \$0.10/minute for each call (a "pay as you go" plan);
- **Plan B:** \$60 flat fee per month (for up to 900 minutes);
- **Plan C:** \$40 flat fee per month, for up to 500 minutes. For every minute over 500, the company charges \$0.40 per minute.

Help the Bell sisters decide which plan is best for them.

Begin by completing the table (you have already completed the columns for **Plan A** and **Plan B**):

Minutes	Cost		
	Plan A	Plan B	Plan C
0	30	60	40
100	40	60	40
200	50	60	40
300	60	60	40
400	70	60	40
500	80	60	40
600	90	60	80
700	100	60	120
800	110	60	160
900	120	60	200
Any number of minutes, $N$	$30+0.10N$	60	$40; N \leq 500$ $40+0.40(N-500); N > 500$

1. Plot the points for **Plan C** on the same graph you created earlier. Use a different symbol to mark **Plan C**, and make a key to show which symbol goes with which plan. See #1 in the Cell Phone Plans activity (above?)
2. Enter the equations you found for each plan using the  $Y=$  button on the graphing calculator. Once you have entered all three equations, press the GRAPH button. To enter the piecewise function generated by **Plan C**, use the following keystrokes:  $(40)(X \leq 500) + (40 + .4(X - 500))(X > 500)$

Note: to enter the  $<$   $>$  symbols, press  $2^{nd}$  **MATH**, and choose the appropriate symbol.

Here is a screen shot of the equations entered after pressing **Y=** :

```

Plot1 Plot2 Plot3
\Y1=30+.1X
\Y2=60
\Y3=(40)(X≤500)+
(40+.4(X-500))(X
>500)
\Y4=
\Y5=
    
```

Be sure to set an appropriate window. Press **WINDOW** and set the following parameters. We want to see the graph from  $x = 0$  to 1000 minutes, and  $y = 0$  to 120 dollars:

```

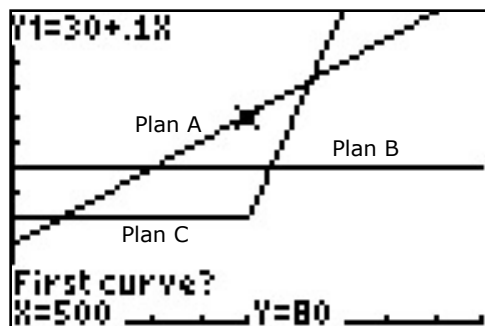
WINDOW
Xmin=0
Xmax=1000
Xscl=100
Ymin=0
Ymax=120
Yscl=10
Xres=1
    
```

3. The three graphs intersect at certain points. How many points of intersection are there? What are the points of intersection? You can use the **CALC** button on the graphing calculator to find the exact values. It is above the **TRACE** button.

**Directions and screen shots from the TI-83 Plus Silver Edition**

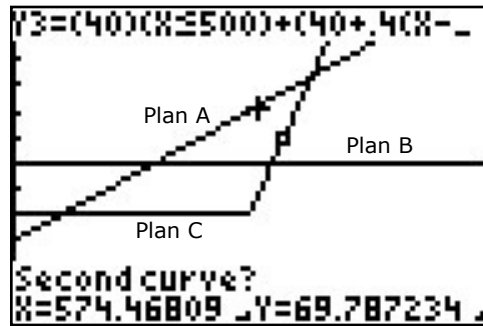
To find the point of intersection between **Plan A** and **Plan C** (Y1 and Y3 in the calculator) using the graphing calculator, follow these steps:

1. After entering the equations in the calculator and setting an appropriate window, press **2<sup>nd</sup> TRACE**, and choose *5: intersect*;
2. The current graph is displayed with *First Curve?* in the bottom left-hand corner (Note: *Plan A*, *Plan B*, and *Plan C* added for clarification. This will NOT show up on the calculator. Also note the equation in the top left-hand corner tells you what graph the cursor is on):

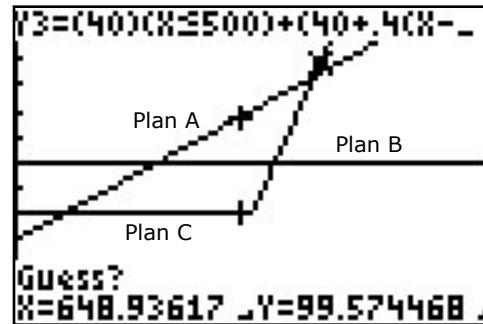


Press the up or down arrow, if necessary, to move the cursor to the first function, and press **ENTER** ;

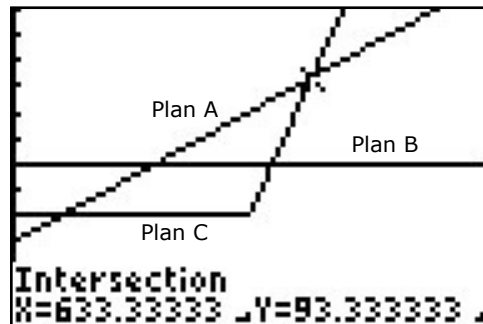
- The current graph is displayed with *Second Curve?* in the bottom left-hand corner:



- Press the up or down arrow, if necessary, to move the cursor to the third function (because we want the intersection between Y1 and Y3) and press **ENTER**;
- Press the right or left arrow key, if necessary, to position the cursor close to the point of intersection, and press **ENTER** :



- The calculator places the cursor on the point of intersection, and the x- and y-values of the intersection are displayed along the bottom of the screen:

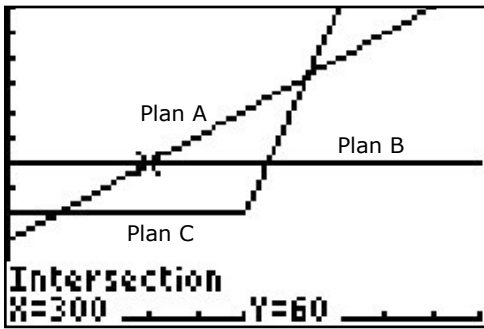


This point of intersection (above) tells us that when  $x = 633 \frac{1}{3}$  minutes, the costs of Plans A and C are equal, at \$93.33/month.

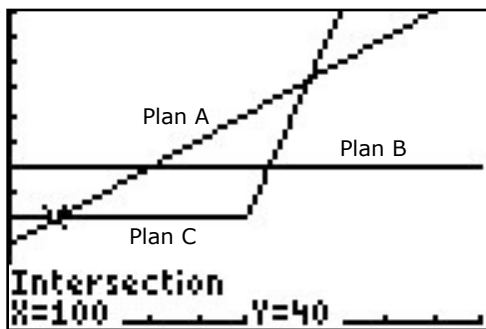
Repeat the steps above to find the other points of intersection.



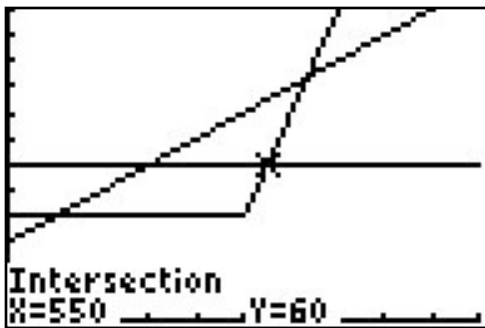
4. Describe what each of these intersections represents in terms of the problem.



This point of intersection (above) tells us that when  $x = 300$  minutes, the costs of Plan A and B are equal, at \$60/month.



This point of intersection (above) tells us that when  $x = 100$  minutes, the costs of Plans A and C are equal, at \$40/month.



Note that Plans A and C have 2 intersection points. This tells us that if  $x > 100$  and  $x < 633 \frac{1}{3}$  minutes, Plan C is less costly. If  $x < 100$  or  $x > 633 \frac{1}{3}$  minutes, Plan A is less costly (excluding Plan B from the analysis). Look for this kind of analysis by students in question 5.

5. Is there a way to use the table to determine the intersection points on the graph? If so, how? You can use the **TABLE** button on the graphing calculator to view the table. It is above the **GRAPH** button.

Yes, the table can be very helpful in locating points of intersection. Look for a row with the same value in each of the Cost columns. For example, at 300 minutes, both **Plan A** and **Plan B** cost \$60. We can also see that **Plan B** and **Plan C** are equal somewhere between 500 and 600 minutes. We can guess they are equal at 550 minutes, and then complete a table with smaller intervals to find the exact point of intersection.

The graphing calculator has a table function that allows the user to see values for each function. Press  $2^{\text{nd}}$  GRAPH to see the table. To set the table starting point and intervals, press  $2^{\text{nd}}$  WINDOW. You will see the screen shot below (left). Enter the starting point in TblStart and the interval in  $\Delta$ Tbl. Set Indpnt and Depend to Auto. After you set the starting values, press  $2^{\text{nd}}$  GRAPH again to see the table. Use the right and left arrow keys to scroll to see all functions (Y1, Y2, and Y3).

Scroll through the table screenshot (right), looking for a row where Y1 and Y2 have the same values. Notice that when  $X = 300$ , both Y1 (Plan A) and Y2 (Plan B) equal \$60. This is the point of intersection between the two functions.

TABLE SETUP		
TblStart=	0	
$\Delta$ Tbl=	100	
Indpnt:	Auto	Ask
Depend:	Auto	Ask

X	Y1	Y2
0	60	30
100	60	40
200	60	50
300	60	60
400	60	70
500	60	80
600	60	90

X=0

To view the table when X is between 500 and 600 minutes, and to change the X interval from 100 to 10 (so that smaller increments of X can be seen), enter the following in TABLE SETUP. Then, press  $2^{\text{nd}}$  GRAPH to get to the table, and using the right arrow key scroll right to see Y3:

TABLE SETUP		
TblStart=	500	
$\Delta$ Tbl=	10	
Indpnt:	Auto	Ask
Depend:	Auto	Ask

X	Y2	Y3
500	60	40
510	60	44
520	60	48
530	60	52
540	60	56
550	60	60
560	60	64

Y3=60

We can now see that Y2 (Plan B) and Y3 (Plan C) have a point of intersection at  $X = 550$  minutes, and the cost of each plan is \$60 (at 550, Plan B and Plan C are equivalent). Look for this kind of analysis from students in Question 5.

- Help the Bell sisters by writing a recommendation for choosing a phone plan. Use data from your graph and table to support your recommendation. Look for students to explain when and/or why each of the three plans would be advisable. Ask questions to help students analyze this scenario, like, "What do you see in the graph/table that leads you to this conclusion?"